Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 27 June 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
6669/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1.

$$\mathbf{A} = \begin{pmatrix} -2 & 1 & -3 \\ k & 1 & 3 \\ 2 & -1 & k \end{pmatrix}, \text{ where } k \text{ is a constant}$$

Given that the matrix **A** is singular, find the possible values of k.

(4)

Leave blank

2. The curve C has equation

$$y = \frac{x^2}{8} - \ln x, \quad 2 \leqslant x \leqslant 3$$

Find the length of the curve C giving your answer in the form $p + \ln q$, where p and q are rational numbers to be found.

(7)

Leave blank

3. (a) Prove that

$$\frac{\mathrm{d}(\operatorname{arcoth} x)}{\mathrm{d}x} = \frac{1}{1 - x^2}$$

(3)

Given that $y = (\operatorname{arcoth} x)^2$,

(b) show that

$$(1 - x^2)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 2x\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{1 - x^2}$$

where k is a constant to be determined.

(5)

Leave blank

Question 3 continued

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4. (i) Find, without using a calculator,

$$\int_3^5 \frac{1}{\sqrt{15 + 2x - x^2}}\,\mathrm{d}x$$

giving your answer as a multiple of π.

(5)

(ii)

(a) Show that

$$5\cosh x - 4\sinh x = \frac{\mathrm{e}^{2x} + 9}{2\mathrm{e}^x}$$

(3)

(b) Hence, using the substitution $u = \mathrm{e}^x$ or otherwise, find

$$\int \frac{1}{5\cosh x - 4\sinh x}\,\mathrm{d}x$$

(4)

Leave blank

Question 4 continued

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5. The hyperbola H has equation

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

The point P $(4\sec\theta, 3\tan\theta)$, $0 < \theta < \frac{\pi}{2}$, lies on H.

(a) Show that an equation of the normal to H at the point P is

$$3y + 4x\sin\theta = 25\tan\theta$$

(5)

The line l is the directrix of H for which $x > 0$

The normal to H at P crosses the line l at the point Q. Given that $\theta = \frac{\pi}{4}$

(b) find the y coordinate of Q, giving your answer in the form $a + b\sqrt{2}$, where a and b are rational numbers to be found.

(6)

Leave blank

Question 5 continued

Leave blank

6. $$\mathbf{M} = \begin{pmatrix} p & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & q \end{pmatrix}$$

where p and q are constants.

Given that $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ is an eigenvector of the matrix **M**,

(a) find the eigenvalue corresponding to this eigenvector,

(3)

(b) find the value of p and the value of q.

(3)

Given that 6 is another eigenvalue of **M**,

(c) find a corresponding eigenvector.

(2)

Given that $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ is a third eigenvector of **M** with eigenvalue 3

(d) find a matrix **P** and a diagonal matrix **D** such that

$$\mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P} = \mathbf{D}$$

(3)

Leave blank

Question 6 continued

Leave blank

7. Given that

$$I_n = \int \frac{\sin nx}{\sin x}\,\mathrm{d}x, \quad n \geqslant 1$$

(a) prove that, for $n \geqslant 3$

$$I_n - I_{n-2} = \int 2\cos(n-1)x\,\mathrm{d}x$$

(3)

(b) Hence, showing each step of your working, find the exact value of

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \frac{\sin 5x}{\sin x}\,\mathrm{d}x$$

giving your answer in the form $\frac{1}{12}\left(a\pi + b\sqrt{3} + c\right)$, where a, b and c are integers to be found.

(7)

Leave blank

8. The plane Π_1 has equation

$$x - 5y - 2z = 3$$

The plane Π_2 has equation

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$$

where λ and μ are scalar parameters.

(a) Show that Π_1 is perpendicular to Π_2

(4)

(b) Find a cartesian equation for Π_2

(2)

(c) Find an equation for the line of intersection of Π_1 and Π_2 giving your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors to be found.

(6)

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Question 8 continued